Chapter 7
Sliding Stability

7-1. Scope

This chapter provides guidance for assessing the sliding stability of laterally loaded structures founded on rock masses. Examples of applicable structures include gravity dams, coffer dams, flood walls, lock walls, and retaining structures. The chapter is divided into three sections to include: modes of failure; methods of analyses; and treatment methods.

Section I
Modes of Failure

7-2. General

Paths along which sliding can occur will be confined to the foundation strata; pass through both the foundation strata and the structure; or just pass through the structure. This chapter addresses sliding where the failure path is confined to the foundation strata or at the interface between the strata and the structure's foundation. Although complex, foundation-structure sliding failure or sliding failure through the structure are conceptually possible and must be checked, such failures are likely to occur only in earth structures (e.g., embankments). The analyses of these later two failure modes are addressed in EM 1110-2-1902.

7-3. Potential Failure Paths

Potential failure paths along which sliding may occur can be divided into five general categories as illustrated in Figure 7-1.

a. Failure along discontinuities. Figure 7-1a illustrates a mode of potential failure where the failure path occurs along an unfavorably oriented discontinuity. The mode of failure is kinematically possible in cases where one or more predominate joint sets strike roughly parallel to the structure and dip in the upstream direction. The case is particularly hazardous with the presence of an additional joint set striking parallel to the structure and dipping downstream. In the absence of the additional joint set, failure is generally initiated by a tensile failure at the heel of the structure. Where possible the structure should be aligned in a manner that will minimize the development of this potential mode of failure.

b. Combined failure. A combined mode of failure is characterized by situations where the failure path can occur both along discontinuities and through intact rock as illustrated in Figure 7-1b. Conceptually, there are any number of possible joint orientations that might result in a combined mode of failure. However, the mode of failure is more likely to occur in geology where the rock is horizontally or near horizontally bedded and the intact rock is weak.

c. Failure along interface. In cases where structures are founded on rock masses containing widely spaced discontinuities, none of which are unfavorably oriented, the potential failure path is likely to coincide with the interface between the structure and the foundation strata. The interface mode of failure is illustrated in Figure 7-1c.

d. Generalized rock mass failure. In the generalized rock mass mode of failure, the failure path is a localized zone of fractured and crushed rock rather than well defined surfaces of discontinuity. As implied in Figure 7-1d, a generalized rock mass failure is more likely to occur in highly fractured rock masses.

e. Buckling failure. Figure 7-1e illustrates a conceptual case where failure is initiated by buckling of the upper layer of rock downstream of the structure. Rock masses conducive to buckling type failures would contain thin, horizontally bedded, rock in which the parent rock is strong and brittle. Although no case histories have been recorded where buckling contributed to or caused failure, the potential for a buckling failure should be addressed where warranted by site conditions.

Section II
Methods of Analysis

7-4. General Approach

The guidance in this chapter is based on conventional geotechnical principles of limit equilibrium. The basic principle of this method applies the factor of safety to the least known conditions affecting sliding stability, this is, the material shear strength. Mathematically, the basic principle is expressed as:

$$\tau = \frac{\tau_f}{FS} \tag{7-1}$$

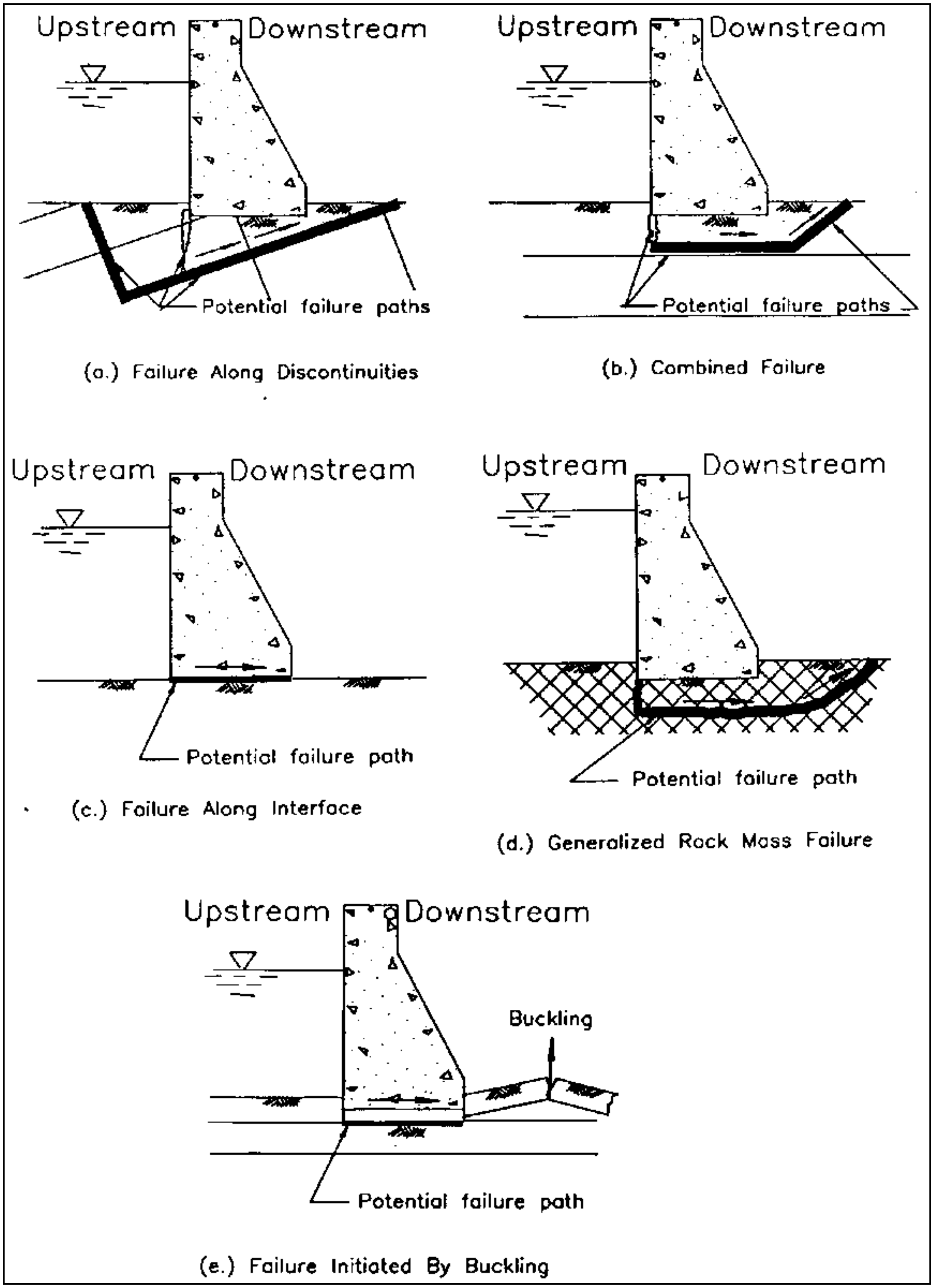

Figure 7-1. Potential failure paths

in which τ is the limiting (applied) shear stress required for equilibrium and τ_f is the maximum available shear strength that can be developed. The ratio of these two quantities, expressed by Equation 7-2, is called the factor of safety.

$$FS = \tau_f / \tau \qquad (7\text{-}2)$$

The maximum available shear strength τ_f is defined by the Mohr-Coulomb failure criterion. Procedures for selecting the appropriate shear strength parameters c and ϕ are discussed in Chapter 4.

7-5. Conditions for Stability

According to this method, the foundation is stable with respect to sliding when, for any potential slip surface, the resultant of the applied shear stresses required for equilibrium is smaller than the maximum shear strength that can be developed. A factor of safety approaching unity for any given potential slip surface implies failure by sliding is impending. The surface along which sliding has the greatest probability of occurring is the surface that results in the smallest factor of safety. This surface is referred to as the potential critical failure surface.

7-6. Assumptions

As in any mathematical expression which attempts to model a geologic phenomenon, the limit equilibrium method requires the imposition of certain simplifying assumptions. Assumptions invariably translate into limitations in application. Limit equilibrium methods will provide an adequate assessment of sliding stability provided that sound engineering judgment is exercised. This judgment requires a fundamental appreciation of the assumptions involved and the resulting limitations imposed. The following discussion emphasizes the more important assumptions and limitations.

a. Failure criterion. Conventional limit equilibrium solutions for assessing sliding stability incorporate the linear Mohr-Coulomb failure criterion (see Figure 4-5) for estimating the maximum available shear strength (τ_f). It is generally recognized that failure envelopes for all modes of rock failure are, as a rule, non-linear. As discussed in Chapter 4, imposition of a linear criterion for failure, as applied to rock, requires experience and judgment in selecting appropriate shear strength parameters.

b. Two-dimensional analysis. The method presented in this chapter is two-dimensional in nature. In most cases, problems associated with sliding in rock masses involve the slippage of three-dimensional wedges isolated by two or more discontinuities and the ground surface. In such cases, a two-dimensional analysis generally results in a conservative assessment of sliding stability. It is possible for a two-dimensional analysis to predict an impending failure where in reality the assumed failure mechanism is kinematically impossible.

c. Failure surface. The stability equations are based on an assumed failure surface consisting of one or more planes. Multiplane surfaces form a series of wedges which are assumed to be rigid. The analysis follows the method of slices approach common to limit equilibrium generalized slip surfaces used in slope stability analysis (e.g., see Janbu 1973). Slices are taken at the intersection of potential failure surface planes. Two restrictions are imposed by the failure surface assumptions. First, the potential failure surface underlying the foundation element is restricted to one plane. Second, planear surfaces are not conducive to search routines to determine the critical potential failure surface. As a result, determination of the critical failure surface may require a large number of trial solutions; particularly in rock masses with multiple, closely spaced, joint sets.

d. Force equilibrium. Equations for assessing stability were developed by resolving applied and available resisting stresses into forces. The following assumptions are made with respect to forces.

(1) Only force equilibrium is satisfied. Moment equilibrium is not considered. Stability with respect to overturning must be determined separately.

(2) In order to simplify the stability equations, forces acting vertically between wedges are assumed to be zero. Neglecting these forces generally results in a conservative assessment of sliding stability.

(3) Because only forces are considered, the effects of stress concentrations are unknown. Potential problems associated with stress concentrations must be addressed separately. The finite element method is ideally suited for this task.

e. Strain compatibility. Considerations regarding displacements are excluded from the limit equilibrium approach. The relative magnitudes of the strain at failure for different foundation materials may influence the results of the sliding stability analysis. Such complex structure-foundation systems may require a more intensive sliding investigation than a limit equilibrium approach. In this respect, the effects of strain compatibility may require special interpretation of data from in-situ tests, laboratory tests, and finite element analyses.

f. Factor of safety. Limit equilibrium solutions for sliding stability assume that the factor of safety of all wedges are equal.

7-7. Analytical Techniques for Multi-Wedge Systems

a. General wedge equations. The general wedge equations are derived from force equilibrium of all wedges in a system of wedges defined by the geometry of the structure and potential failure surfaces. Consider the ith wedge in a system of wedges illustrated in Figure 7-2. The necessary geometry notation for the ith wedge and adjacent wedges are as shown (Figure 7-2). The origin of the coordinate system for the wedge considered is located in the lower left hand corner of the wedge. The x and y axes are horizontal and vertical respectively. Axes which are tangent (t) and normal (n) to the failure plane are oriented at an angle (α) with respect to the $+x$ and $+y$ axes. A positive value of α is a counterclockwise rotation, a negative value of α is a clockwise rotation. The distribution of pressures/stresses with resulting forces is illustrated in Figure 7-3. Figure 7-4 illustrates the free body diagram of the resulting forces. Summing the forces normal and tangent to the potential failure surface and solving for (P_{i-1} - P_i) results in the following equation for the ith wedge:

$$
\begin{aligned}
(P_{i-1} - P_i) = \Big[& ((W_i + V_i)\cos\alpha_i \\
& - U_i + (H_{Li} - H_{Ri})\sin\alpha_i) \\
& \frac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i \\
& + (W_i + V_i)\sin_{\alpha_i} + \frac{C_i}{FS_i}L_i \Big] \\
& \div \left[\cos\alpha_i - \sin\alpha_i \frac{\tan\phi_i}{FS_i} \right]
\end{aligned}
\tag{7-3}
$$

where

i = subscript notation for the wedge considered

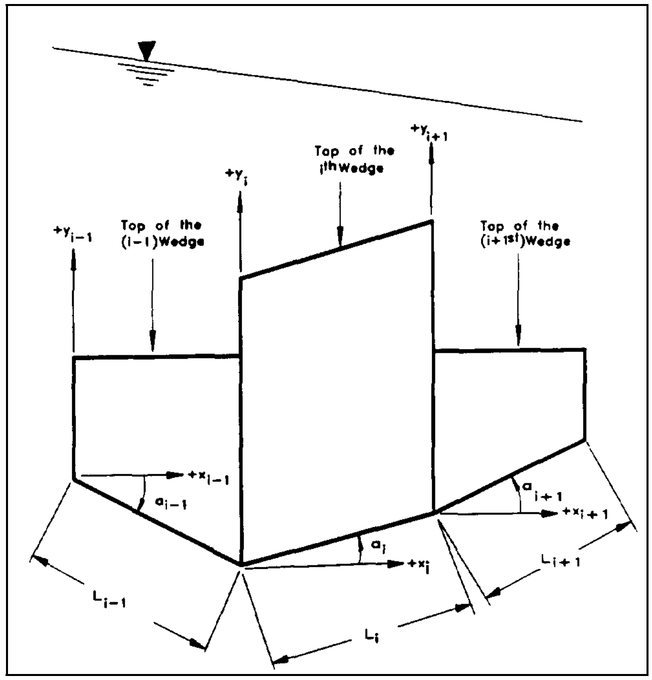

Figure 7-2. Hypothetical i^th wedge and adjacent wedges subject to potential sliding

P = horizontal residual forces acting between wedges as a result of potential sliding

W = the total weight of wedge to include rock, soil, concrete and water (do not use submerged weights)

V = any vertical force applied to the wedge

α = angle of potential failure plane with respect to the horizontal ($-\alpha$ denotes downslope sliding, $+\alpha$ denotes upslope sliding)

U = the uplift force exerted on the wedge at the potential failure surface

H = in general, any horizontal force applied to the wedge (H_L and H_R refers to left and right hard forces as indicated in Figures 7-3 and 7-4)

L = the length of the wedge along the potential failure surface

FS = the factor of safety

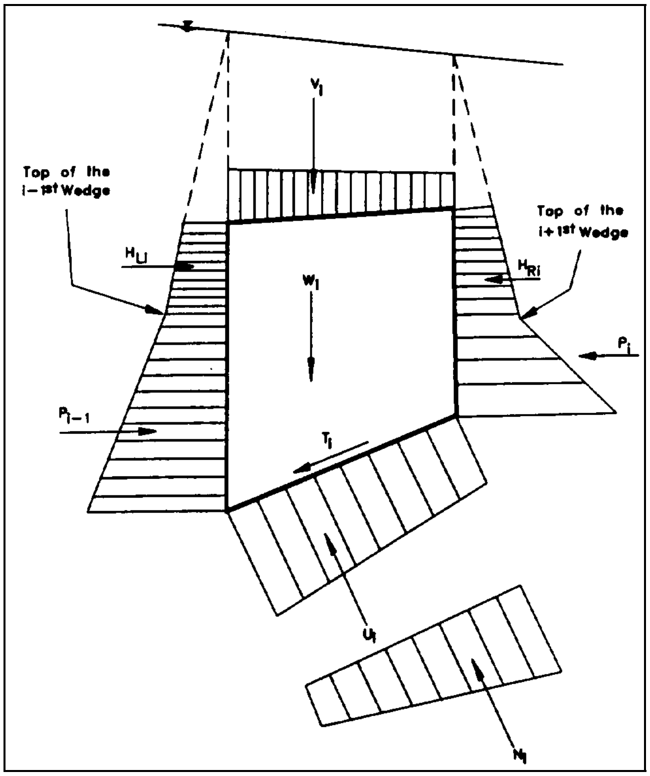

Figure 7-3. Distribution of pressures, stresses and resultant forces acting on the hypothetical i^th wedge

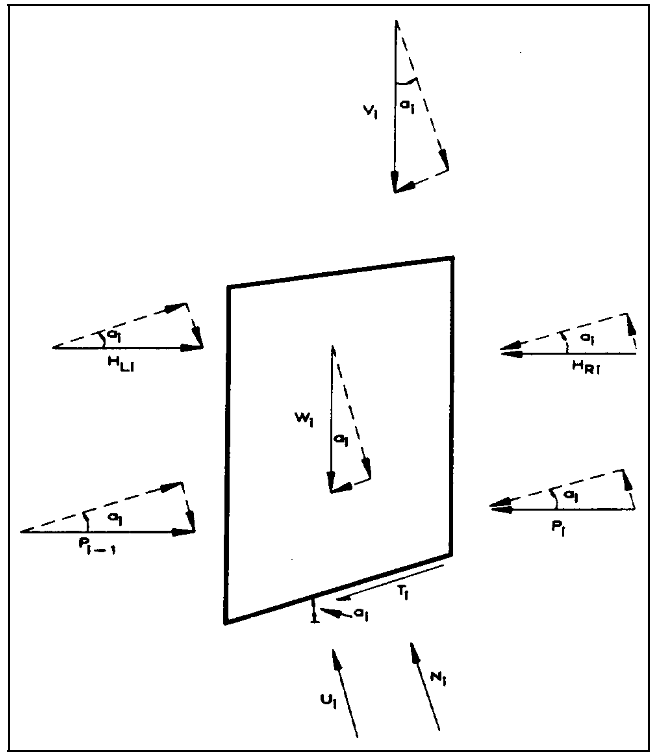

Figure 7-4. Free body diagram of the hypothetical i^th wedge

c = the cohesion shear strength parameter

ϕ = the angle of internal friction

b. Equilibrium requirements. An inspection of Equation 7-3 reveals that for a given wedge there will be two unknowns (i.e., $(P_{i-1} - P_i)$ and FS). In a wedge system with n number of wedges, Equation 7-3 will provide n number of equations. Because FS is the same for all wedges there will be $n + 1$ unknowns with n number of equations for solution. The solution for the factor of safety is made possible by a conditional equation establishing horizontal equilibrium of the wedge system. This equation states that the sum of the differences in horizontal residual forces $(P_{i-1} - P_i)$ acting between wedges must equal the differences in the horizontal boundary forces. Since boundary forces are usually equal to zero, the conditional equation is expressed as

$$\sum_{i=1}^{i=n} (P_{i-1} - P_i) = 0 \tag{7-4}$$

where

n = the total number of wedges in the system.

c. Alternate equation. An alternate equation for the implicit solution of the factor of safety for a system of n wedges is given below:

$$FS = \frac{\sum_{i=1}^{i=n} \dfrac{C_i L_i \cos\alpha_i + (W_i + V_i - U_i \cos\alpha_i)\tan\phi_i}{n_{\alpha_i}}}{\sum_{i=1}^{i=n} [H_i - (W_i + V_i)\tan\alpha_i]} \tag{7-5a}$$

where

$$n_{\alpha_i} = \frac{1 - \dfrac{\tan\phi_i \tan\alpha_i}{FS}}{1 + \tan^2\alpha_i} \tag{7-5b}$$

All other terms are as defined above. The derivation of Equations 7-5 follows that of Equations 7-3 and 7-4 except that forces are summed with respect to the x and y coordinates.

7-8. Preliminary Procedures

Factor of safety solutions for a multi-wedge system containing a number of potential failure surfaces can result in a significant book-keeping problem. For this reason, it is recommended that prior to the analytical solution for the factor of safety, the following preliminary procedures be implemented.

a. Define and identify on a scale drawing all potential failure surfaces based on the stratification, location, orientation, frequency, and distribution of discontinuities within the foundation material as well as the geometry, location, and orientation of the structure.

b. For each potential failure surface, divide the mass into a number of wedges. A wedge must be created each time there is a change in slip plane orientation and/or a change in shear strength properties. However, there can be only one structural wedge.

c. For each wedge draw a free body diagram which shows all the applied and resulting forces acting on that wedge. Include all necessary dimensions on the free body diagram. Label all forces and dimensions according to the appropriate parameter notations discussed above.

d. Prepare a table, which lists all parameters, to include shear strength parameters for each wedge in the system of wedges defining the potential slip mass.

7-9. Analytical Procedures

While both the general wedge equation and the alternate equation will result in the same calculated factor of safety for a given design case, the procedure for calculating that value is slightly different. Solutions for hypothetical example problems are provided in EM 1110-2-2200 and Nicholson (1983a).

a. General wedge method. The solution for the factor of safety using Equations 7-3 and 7-4 requires a trial-and-error procedure. A trial value for the factor of safety, *FS*, is inserted in Equation 7-3 for each wedge to obtain values of the differences in horizontal residual *P* forces acting between wedges. The differences in *P* forces for each wedge are then summed; a negative value indicates that the trial value of *FS* was to high and conversely a positive value indicates that the trial value of *FS* was too low. The process is repeated until the trial *FS* value results in an equality from Equation 7-4. The value of *FS* which results in an equality is the correct value for the factor of safety. The number of trial-and-error cycles can be reduced if trial values of *FS* are plotted with respect to the sum of the differences of the *P* forces (see examples in EM 1110-2-2200 and Nicholson (1983a)).

b. Alternate methods. Equations 7-5a and 7-5b, when expanded, can be used to solve for the factor of safety for a system containing one or more wedges. Since the n_α term, defined by Equation 7-5b, is a function of *FS*, the solution for *FS* requires an iterative process. An assumed initial value of *FS* is inserted into the n_α term for each wedge in the expanded form of Equation 7-5a, and a new factor of safety is calculated. The calculated factor of safety is then inserted into the n_α term. The process is repeated until the inserted value of *FS* equals the calculated value of *FS*. Convergence to within two decimal places usually occurs in 3 to 4 iteration cycles.

c. Comparison of methods. The general wedge equation (Equation 7-3) was formulated in terms of the difference in horizontal boundary forces to allow the design engineer to solve directly for forces acting on the structure for various selected factors of safety. The procedure has an advantage for new structures in that it allows a rapid assessment of the horizontal forces necessary for equilibrium for prescribed factors of safety. The alternate equation (Equation 7-5a and 7-5b) solves directly for *FS*. Its advantage is in the assessment of stability for existing structures. Both equations are mathematically identical (Nicholson 1983a).

7-10. Design Considerations

Some special considerations for applying the general wedge equation to specific site conditions are discussed below.

a. Active wedge. The interface between the group of active wedges and the structural wedge is assumed to be a vertical plane located at the heel of the structural wedge and extending to the base of the structural wedge. The magnitudes of the active forces depend on the actual values of the safety factor, the inclination angles (α) of the slip path, and the magnitude of the shear strength that can be developed. The inclination angles, corresponding to the maximum active residual *P* forces for each potential failure surface, can be determined by independently analyzing the group of active wedges for trial safety factors. In rock the inclination may be predetermined by discontinuities in the foundation.

b. Structural wedge. Discontinuities in the slip path beneath the structural wedge should be modeled by assuming an average slip-plane along the base of the structural wedge.

c. Passive wedge. The interface between the group of passive wedges and the structural wedge is assumed to be a vertical plane located at the toe of the structural wedge and extending to the base of the structural wedge. The magnitudes of the passive residual *P* forces depend on the actual values of the safety factor, the inclination angles of the slip path, and the magnitude of shear strength that can be developed. The inclination angles, corresponding to the minimum passive residual *P* forces for each potential failure mechanism, can be estimated by independently analyzing the group of passive wedges for trial safety factors. When passive resistance is used special considerations must be made. Removal of the passive wedge by future construction must be prevented. Rock that may be subjected to high velocity water scouring should not be used unless amply protected. Also, the compressive strength of the rock layers must be sufficient to develop the wedge resistance. In some cases wedge resistance should not be assumed without resorting to special treatment such as installing rock anchors.

d. Tension cracks. Sliding analyses should consider the effects of cracks on the active side of the structural wedge in the foundation material due to differential settlement, shrinkage, or joints in a rock mass. The depth of cracking in cohesive foundation material can be estimated in accordance with the following equations.

$$d_c = \frac{2c_d}{\gamma}\tan\left(45 - \frac{\phi_d}{2}\right) \tag{7-6a}$$

where

$$c_d = \frac{c}{FS} \tag{7-6b}$$

$$\phi_d = \tan^{-1}\left(\frac{\tan\phi}{\mathrm{FS}}\right) \tag{7-6c}$$

The value (d_c) in a cohesive foundation cannot exceed the embedment of the structural wedge. The depth of cracking in massive, strong, rock foundations should be assumed to extend to the base of the structural wedge. Shearing resistance along the crack should be ignored and full hydrostatic pressure should be assumed to extend to the bottom of the crack. The hydraulic gradient across the base of the structural wedge should reflect the presence of a crack at the heel of the structural wedge.

e. Uplift without drains. The effects of seepage forces should be included in the sliding analysis. Analyses should be based on conservative estimates of uplift pressures. Estimates of uplift pressures on the wedges can be based on the following assumptions:

(1) The uplift pressure acts over the entire area of the base.

(2) If seepage from headwater to tailwater can occur across a structure, the pressure head at any point should reflect the head loss due to water flowing through a medium. The approximate pressure head at any point can be determined by the line-of-seepage method. This method assumes that the head loss is directly proportional to the length of the seepage path. The seepage path for the structural wedge extends from the upper surface (or internal ground-water level) of the uncracked material adjacent to the heel of the structure, along the embedded perimeter of the structural wedge, to the upper surface (or internal ground-water level) adjacentto the toe of the structure. Referring to Figure 7-5, the seepage distance is defined by points a, b, c, and d. The pressure head at any point is equal to the elevation head minus the product of the hydraulic gradient times the distance along the seepage path to the point in question. Estimates of pressure heads for the active and passive wedges should be consistent with those of the heel and toe of the structural wedge.

(3) For a more detailed discussion of the line-of-seepage method, refer to EM 1110-2-2502, Retaining and Flood Walls. For the majority of structural stability computations, the line-of-seepage is considered sufficiently accurate. However, there may be special situations where the flow net method is required to evaluate seepage problems.

f. Uplift with drains. Uplift pressures on the base of the structural wedge can be reduced by foundation drains. The pressure heads beneath the structural wedge developed from the line-of-seepage analysis should be modified to reflect the effects of the foundation drains. The maximum pressure head along the line of foundation drains can be estimated from Equation 7-7:

$$U_x = U_1 + R\left(\frac{L - x}{L}\right)(U_2 - U_1) \tag{7-7}$$

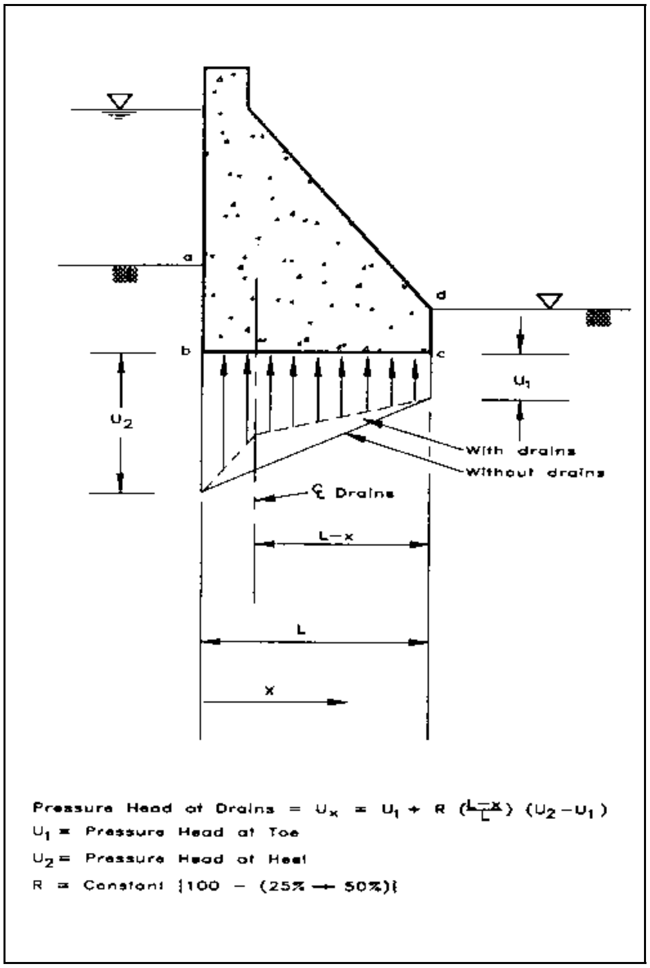

Figure 7-5. Uplift pressures

All parameters are defined in Figure 7-5. The uplift pressure across the base of the structural wedge usually varies from the undrained pressure head at the heel to the assumed reduced pressure head at the line of drains to the undrained pressure head at the toe, as shown in Figure 7-5. Uplift forces used for the sliding analyses should be selected in consideration of conditions which are presented in the applicable design memoranda. For a more detailed discussion of uplift under gravity dams, refer to EM 1110-2-2200, Gravity Dams.

g. Overturning. As stated previously, requirements for rotational equilibrium are not directly included in the general sliding stability equations. For some load cases, the vertical component of the resultant load will lie outside the kern of the base area, and a portion of the structural wedge will not be in contact with the foundation material. The sliding analysis should be modified for these load cases to reflect the following secondary effects due to coupling of sliding and overturning behavior.

(1) The uplift pressure on the portion of the base which is not in contact with the foundation material should be a uniform value which is equal to the maximum value of the hydraulic pressure across the base (except for instantaneous loads such as those due to seismic forces).

(2) The cohesive component of the sliding resistance should only include the portion of the base area which is in contact with the foundation material.

(3) The resultant of the lateral earth (soil) pressure is assumed to act at 0.38 of the wall height for horizontal or downward sloping backfills and at 0.45 of the wall height for upward sloping backfills.

(4) Cantilever or gravity walls on rock should be designed for at-rest earth pressures unless the foundation rock has an unusually low modulus.

7-11. Seismic Sliding Stability

The sliding stability of a structure for an earthquake-induced base motion should be checked by assuming the specified horizontal earthquake acceleration coefficient and the vertical earthquake acceleration coefficient, if included in the analysis, to act in the most unfavorable direction. The earthquake-induced forces on the structure and foundation wedges may then be determined by a quasi-static rigid body analysis. For the quasi-static rigid body analysis, the horizontal and vertical forces on the structure and foundation wedges may be determined by using the following equations:

$$H_{di} = M_i \ddot{X} + m_i \ddot{X} + H_i \tag{7-8}$$

$$V_{di} = M_i g - m_i \ddot{y} \tag{7-9}$$

where

H_d = horizontal forces acting on the structure and/or wedge

V_d = vertical forces acting on the structure and or wedge

M = mass of the structure and/or wedge (weight/g)

m = added mass of reservoir and/or adjacent soil/rock

g = acceleration of gravity

$\ddot{X}$ = horizontal earthquake acceleration coefficient

$\ddot{y}$ = vertical earthquake acceleration coefficient

The subscript i, H, and V terms are as defined previously.

a. Earthquake acceleration. The horizontal earthquake acceleration coefficient can be obtained from seismic zone maps (ER 1110-2-1806) or, in the case where a design earthquake has been specified for the structure, an acceleration developed from analysis of the design earthquake. Guidance is being prepared for the latter type of analysis and will be issued in the near future; until then, the seismic coefficient method is the most expedient method to use. The vertical earthquake acceleration is normally neglected but can be taken as two-thirds of the horizontal acceleration if included in the analysis.

b. Added mass. The added mass of the reservoir and soil can be approximated by Westergaard's parabola (EM 1110-2-2200) and the Mononobe-Okabe method (EM 1110-2-2502), respectively. The structure should be designed for a simultaneous increase in force on one side and decrease on the opposite side of the structure when such can occur.

c. Analytical procedures. The analytical procedures for the seismic quasi-static analyses follows the procedures outlined in paragraphs 7-9*a* and 7-9*b* for the general wedge and alternate methods, respectively. However, the H_d and V_d terms are substituted for the H and W terms, respectively, in Equations 7-3 and 7-5a.

7-12. Factor of Safety

For major concrete structures (dams, lockwalls, basin walls which retain a dam embankment, etc.) the minimum required factor of safety for normal static loading conditions is 2.0. The minimum required factor of safety for seismic loading conditions is 1.3. Retaining walls on rock require a safety factor of 1.5; refer to EM 1110-2-2502 for a discussion of safety factors for floodwalls. Any relaxation of these values will be allowed only with the approval of CECW-E and should be justified by comprehensive foundation studies of such nature as to reduce uncertainties to a minimum.

Section III
Treatment Methods

7-13. General

Frequently a sliding stability assessment of structures subjected to lateral loading results in an unacceptably low factor of safety. In such cases, a number of methods are available for increasing the resistance to sliding. An increase in sliding resistance may be achieved by one or a combination of three mechanistic provisions. The three provisions include: increasing the resisting shear strength by increasing the stress acting normal to the potential failure surface; increasing the passive wedge resistance; and providing lateral restraining forces.

7-14. Increase in Shear Strength

The shear strength available to resist sliding is proportional to the magnitude of the applied stress acting normal to the potential slip surface. An increase in the normal stress may be achieved by either increasing the vertical load applied to the structural wedge and/or passive wedge(s) or by a reduction in uplift forces. The applied vertical load can be conveniently increased by increasing the mass of the structure or placing a berm on the downstream passive wedge(s). Installation of foundation drains and/or relief wells to relieve uplift forces is one of the most effective methods by which the stability of a gravity hydraulic structure can be increased.

7-15. Increase in Passive Wedge Resistance

Resistance to sliding is directly influenced by the size of the passive wedge acting at the toe of the structure. The passive wedge may be increased by increasing the depth the structure is embedded in the foundation rock or by construction of a key. Embedment and keys are also effective in transferring the shear stress to deeper and frequently more competent rock.

7-16. Lateral Restraint

Rock anchors inclined in the direction of the applied shear load provide a force component which acts against the applied shear load. Guidance for the design of anchor systems is discussed in Chapter 9 of this manual.